

# 0 Butterworth Filters

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## Objectives

1. To use basic filter circuits to design multi-order filter response.
2. To recognize the interchangeability of components to alter design and create high pass or low pass filters.
3. To apply Butterworth polynomial coefficients in the design of filters.
4. To control operational amplifier gain in filter circuits where the gain is often critical to proper design.
5. To cascade filters to create different characteristics than exist with the individual filters.
6. To simulate various multi-order filters using Electronics Workbench™ software running in a Macintosh™ or Windows™ environment, or other similar software.

## Equipment

- 8 Resistors: 680 (2), 3.3 k (2), 5.6 k (2), 68 k (2)
- 4 Capacitors: 47 nF (4)
- 2 LM741 Operational Amplifiers

## Information

There are a number of different multi-order filter circuit configurations, one of which is Butterworth. A multi-order filter is one which has slope rates exceeding  $\pm 20$  dB/decade. A third order filter, for example, will have a slope rate of  $\pm 60$  dB/decade. At the filter break frequency or critical frequency,  $f_c$ , the filter response may be peaked or attenuated, depending on the circuit damping factor, zeta ( $\zeta$ ). The effects of  $\zeta$  will not be considered in this experiment as  $\zeta$  will be set to 0.707, thus providing -3 dB response at the critical frequency. A later experiment may consider the effects of  $\zeta$  separately.

Second order Butterworth filters may be designed using the circuit of Figure 0-1, illustrated as a low pass filter. Interchanging the positions of both  $R$ s with both  $C$ s will result in a high pass filter at the same critical frequency. In either case, Equation 0-1 yields the critical frequency,  $f_c$ .

$$f_c = \frac{1}{2\pi RC} \dots \dots \dots \text{Equation 0-1}$$

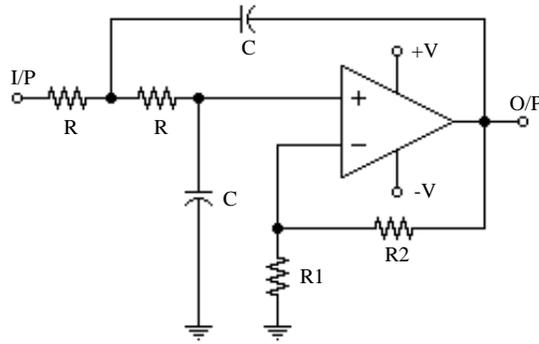


Figure 0-1. Second Order Low Pass Butterworth Filter

The circuit of Figure 0-2, a first order low pass filter with  $f_c$  also as described by Equation 0-1, could be added to the output of the circuit of Figure 0-1 to create a third order filter having a slope of -60 dB/decade. However, the critical frequency where the response is -3 dB will be affected by the interaction between the two filters.

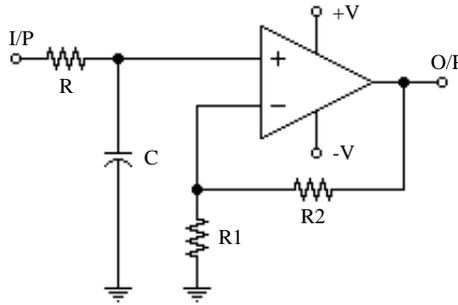


Figure 0-2. First Order Low Pass Filter

Any multi-order Butterworth filter can be designed by using normalized Butterworth polynomials presented in Table 0-1 and by using combinations of the circuits of Figures 0-1 and 0-2. A fourth order filter would cascade two second order filters, a fifth order filter would cascade two second order filters with one first order filter. In most of these cases, stage gain values are critical to obtaining expected break frequencies.

Table 0-1. Butterworth Polynomials

Filter Order	Polynomial Factors
1	$(s+1)$
2	$(s^2+1.414s+1)$
3	$(s+1) (s^2+1.000s+1)$
4	$(s^2+1.848s+1) (s^2+0.765s+1)$
5	$(s+1) (s^2+1.618s+1) (s^2+0.618s+1)$
6	$(s^2+1.932s+1) (s^2+1.414s+1) (s^2+0.5176s+1)$
7	$(s+1) (s^2+1.802s+1) (s^2+1.247s+1) (s^2+0.4450s+1)$
8	$(s^2+1.962s+1) (s^2+1.663s+1) (s^2+1.111s+1) (s^2+0.3902s+1)$

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To utilize the Butterworth Polynomials for filter design, it must be realized that the “(s+1)” response is realized with a first order filter such as Figure 0-2, and the “(s<sup>2</sup>+1.414s+1)” response is realized with a second order filter such as Figure 0-1. The required gain for each second order stage is determined by the coefficient of s, for which we use the symbol  $\zeta$ . To assure the -3 dB filter response occurs at  $f_c$ , the second order stage voltage gain is set using Equation 0-2. The stage gain for the first order stages is arbitrary, which means that it may be set for any value. The filter response at the critical frequency is determined by the damping factor,  $\zeta$ , set to 0.707 in this experiment. If a different damping factor is required, the relationship between  $\zeta$  and  $Q$  is described in Equation 0-3.

$$A_v = 3 - \dots \dots \dots \text{Equation 0-2}$$

$$= 2 \dots \dots \dots \text{Equation 0-3}$$

To illustrate the use of the Butterworth Polynomials, assume we require a fifth order low pass filter with  $f_c = 4.0$  kHz. Table 0-1 indicates we will need a first order filter and two second order filters. Many combinations of R and C will provide  $f_c = 4.0$  kHz; the trick is to find available component values which will be close. If we select a 10 nF capacitor, Equation 0-1 yields a value for R of 3.98 k for which we could select a commonly-available resistor value of 3.9 k. For all three circuits then  $R = 3.9$  k and  $C = 10$  nF.

The gain of the first order stage is arbitrary so if we selected a gain of 2.0, we could use 1 k resistors for R1 and R2 of the first order stage. Table 0-1 indicates a coefficient of  $\zeta = 1.618$  so Equation 0-2 indicates a required gain of  $3 - 1.618$ , or 1.382 for one of the second order stages. Table 0-1 indicates a coefficient of  $\zeta = 0.6180$  so Equation 0-2 indicates a required gain of  $3 - 0.6180$ , or 2.382 for the other second order stage. The non-inverting gain of the op amps is  $(R1+R2)/R1$ , therefore if a value for R1 is selected, then  $R2 = R1(A_v-1)$ . To obtain the gain of 1.382, select  $R1 = 1$  k, then  $R2 = 1$  k  $(2.382-1) = 1.382$  k which could be made with 1.2 k in series with 180. To obtain the gain of 2.382, select  $R1 = 1$  k, then  $R2 = 1$  k  $(1.382-1) = 382$  for which a 390 resistor might be selected. Other combinations of R1 and R2 might be selected to provide closer results. The resulting design is illustrated in Figure 0-3.

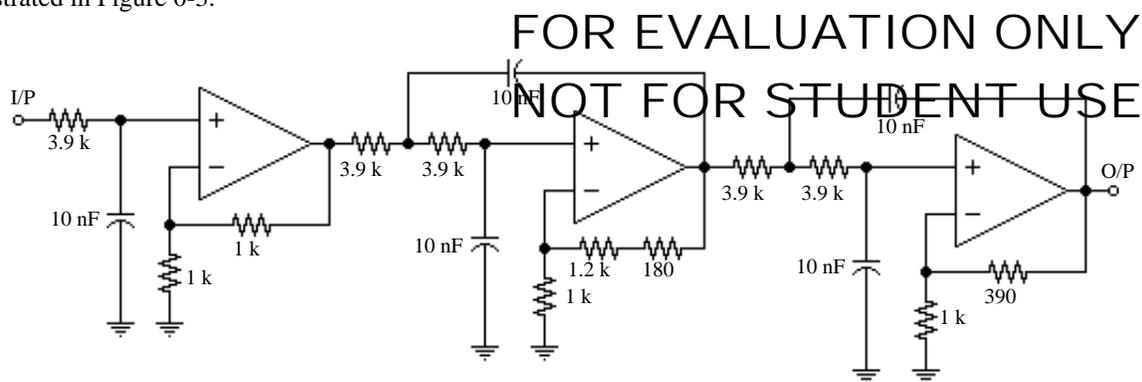


Figure 0-3. A Fifth Order Low Pass Filter

This experiment will investigate a second order low pass Butterworth filter with  $f_c = 5.0$  kHz and a second order high pass Butterworth filter with  $f_c = 50$  Hz. After both circuits are working, they will be cascaded to produce a 50 Hz to 5.0 kHz band pass filter.



## Lab Preparation

1. Design a second order Butterworth low pass filter to provide  $f_c = 5.00$  kHz, using an LM741 op amp. Determine all component values and draw the completed schematic diagram on the Lab Prep summary sheet.
2. Design a second order Butterworth high pass filter to provide  $f_c = 50.0$  Hz, using an LM741 op amp. Determine all component values and draw the completed schematic diagram on the Lab Prep summary sheet.
3. If circuit simulation software is available, simulate both of the above circuits and run a frequency response graph to display the gain versus log frequency characteristic of each circuit.

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## Procedure

### Second Order Low Pass Butterworth Filter

1. You may be required to build either the circuit of Figure 0-4 or the circuit which you designed in the Lab Preparation section. Ask your instructor which circuit you are to build.

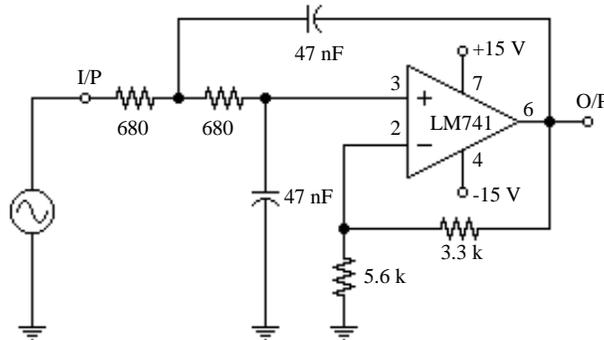


Figure 0-4. Low Pass Circuit

2. Apply appropriate voltages to the input of the circuit of Figure 0-4 so as to complete the measurements of Table 0-2. Be sure the op amp output is not overdriven.
3. Do the required calculations in Table 0-2 and plot the results on the graph provided on the Data Summary sheet. Leave your circuit connected for further use.

Table 0-2. Low Pass Measurements

Frequencies	Measurements		Calculations	
	$V_i$	$V_o$	$A_v$	$A_v(\text{dB})$
100 Hz				
1.00 kHz				
2.00 kHz				
4.00 kHz				
5.00 kHz				
6.00 kHz				
10.0 kHz				
20.0 kHz				
40.0 kHz				

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## Second Order High Pass Butterworth Filter

- You may be required to build either the circuit of Figure 0-5 or the circuit which you designed in the Lab Preparation section. Ask your instructor which circuit you are to build.

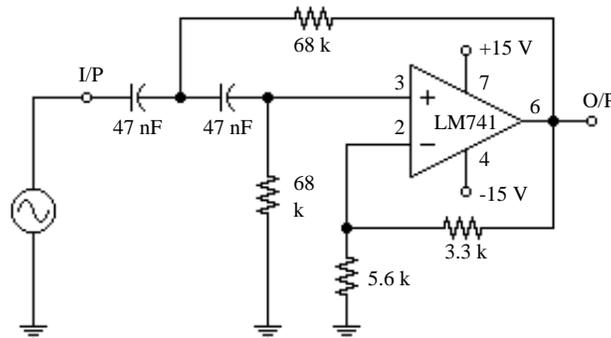


Figure 0-5. High Pass Circuit

- Apply appropriate voltages to the input of the circuit of Figure 0-5 so as to complete the measurements of Table 0-3. Be sure the op amp output is not overdriven.
- Do the required calculations in Table 0-3 and plot the results on the graph provided on the Data Summary sheet. Leave your circuit connected for further use.

Table 0-3. High Pass Measurements

Frequencies	Measurements		Calculations	
	$V_i$	$V_o$	$A_v$	$A_{v(dB)}$
1.00 kHz				
500 Hz				
200 Hz				
100 Hz				
60 Hz	FOR EVALUATION ONLY NOT FOR STUDENT USE			
50 Hz				
40 Hz				
20 Hz				
10 Hz				

## Second Order Band Pass Butterworth Filter

7. Cascade the low pass and high pass filter circuits together as shown in Figure 0-6.

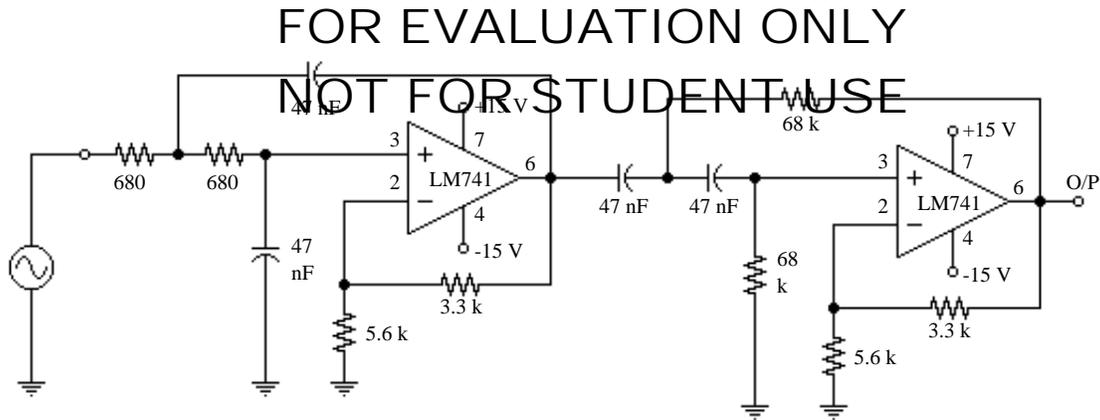


Figure 0-6. Band Pass Circuit

8. Apply appropriate voltages to the input of the circuit of Figure 0-6 so as to complete the measurements of Table 0-4. Be sure the op amp output is not overdriven.
9. Do the required calculations in Table 0-4 and plot the results on the graph provided on the Data Summary sheet.

Table 0-4. Band Pass Measurements

Frequencies	Measurements		Calculations	
	$V_i$	$V_o$	$A_v$	$A_{v(dB)}$
10 Hz				
20 Hz				
40 Hz				
50 Hz				
60 Hz				
100 Hz	FOR EVALUATION ONLY NOT FOR STUDENT USE			
200 Hz				
500 Hz				
1 kHz				
2 kHz				
4 kHz				
5 kHz				
6 kHz				
10 kHz				
20 kHz				
40 kHz				



1. Draw the completed design for the low pass Butterworth filter having  $f_c = 5.00$  kHz in the space below.

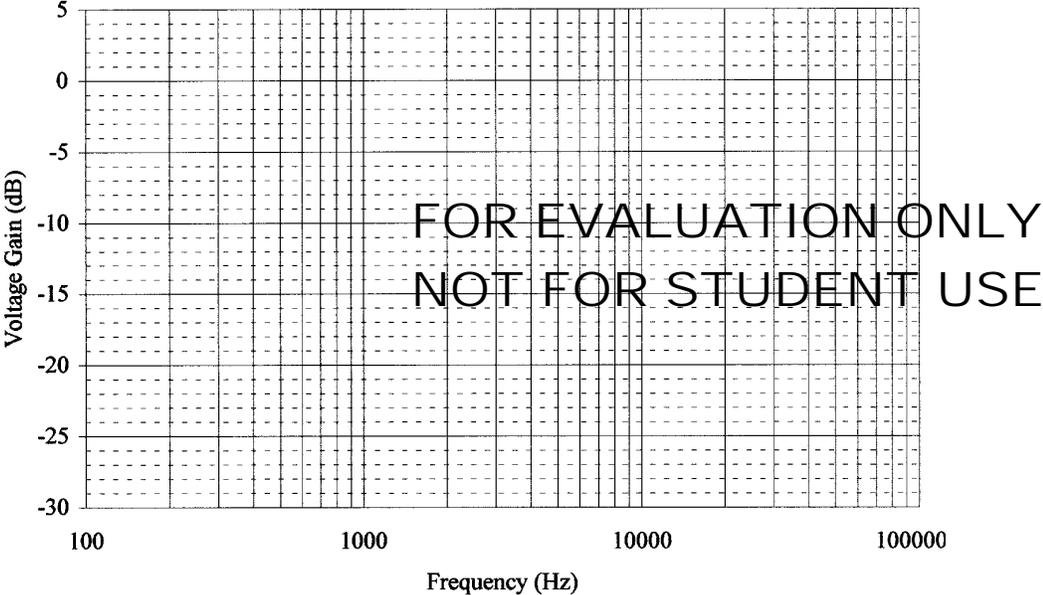
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2. Draw the completed design for the high pass Butterworth filter having  $f_c = 50.0$  Hz in the space below.

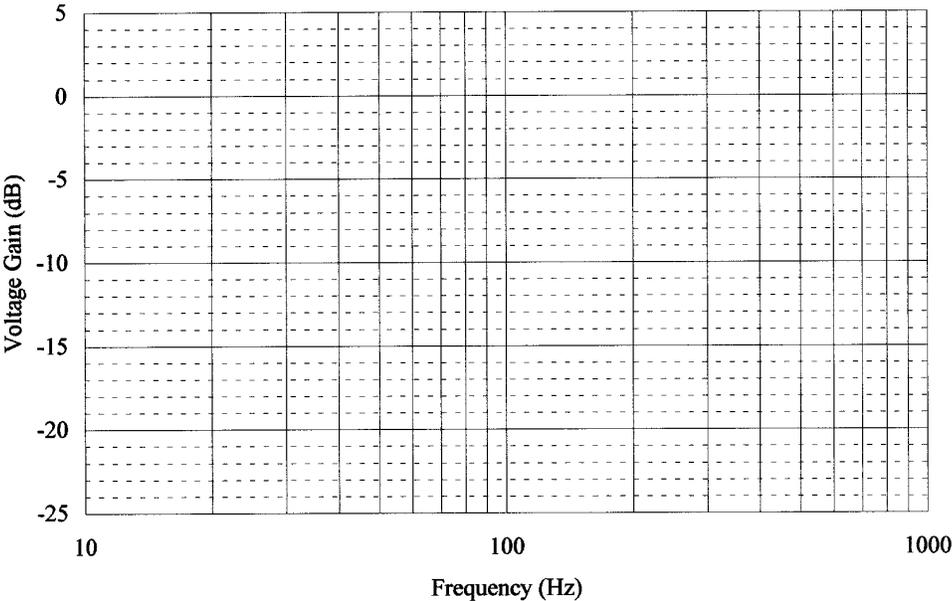
3. Circuit simulation results may be placed on the other side of this sheet.



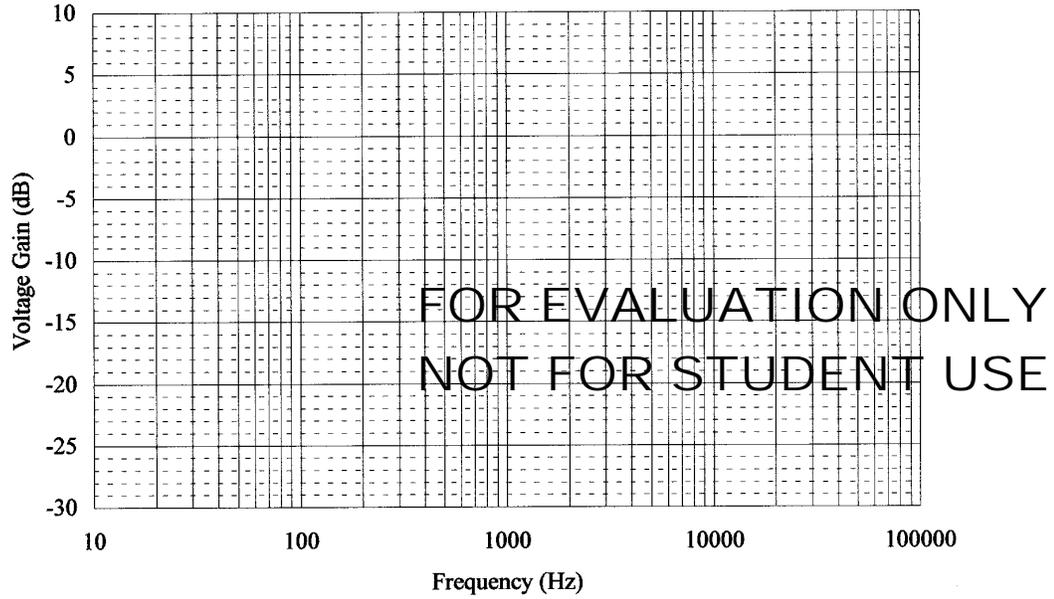
- 1. Complete Graph 0-1 below for the low pass filter using the data from Table 0-2.



- 2. Complete Graph 0-2 below for the high pass filter using the data from Table 0-3.



3. Complete Graph 0-3 below for the band pass filter using the data from Table 0-4.



## Questions

1. From your graphical or tabular data for the low pass filter, determine the measured rolloff slope and  $f_c$ .
2. From your graphical or tabular data for the high pass filter, determine the measured rolloff slope and  $f_c$ .
3. From your graphical or tabular data for the band pass filter, determine the measured rolloff slopes and  $f_c$ 's.